# Trigonometry

# Trigonometric

Today we will be covering a very important topic from the Advance Maths part of the Quantitative Aptitude section that is – Important Notes & Short Tricks on Trigonometric Identities.

Important Short Tricks on Trigonometric Identities

### Pythagorean Identities

- $\sin^2\theta + \cos^2\theta = 1$
- $\tan^2\theta + 1 = \sec^2\theta$
- $\cot^2\theta + 1 = \csc^2\theta$

## Negative of a Function

- sin (–x) = –sin x
- cos (–x) = cos x
- tan (–x) = –tan x
- csc (–x) = –csc x
- sec (–x) = sec x
- cot (–x) = –cot x

## If A + B = 90o, Then

- Sin A = Cos B
- $\text{Sin}^2 A + \text{Sin}^2 B = \text{Cos}^2 A + \text{Cos}^2 B = 1$
- Tan A = Cot B
- Sec A = Csc B

# Trigonometry

## For example:

If tan (x+y) tan (x-y) = 1, then find tan (2x/3)?

## Solution:

Tan A = Cot B, Tan A*Tan B = 1

So, A +B = $90^{o}$

(x+y)+(x-y) = $90^{o}$, 2x = $90^{o}$ , x = $45^{o}$

Tan (2x/3) = tan $30^{o}$= $1/\sqrt{3}$

## If A – B = $90^{o}$, (A › B) Then

- Sin A = Cos B
- Cos A = – Sin B
- Tan A = – Cot B

## If A ± B = $180^{o}$, then

- Sin A = Sin B
- Cos A = – Cos B

## If A + B = $180^{o}$

Then, tan A = – tan B

# Trigonometry

## If A – B = $180^o$

Then, tan A = tan B

## For example:

Find the Value of tan $80^o$ + tan 100o ?

**Solution:** Since 80 + 100 = 180

Therefore, tan $80^o$ + tan 100o = 1

## If A + B + C = $180^o$, then

Tan A + Tan B +Tan C = Tan A * Tan B *Tan C

sin θ * sin 2θ * sin 4θ = ¼ sin 3θ

cos θ * cos 2θ * cos 4θ = ¼ cos 3θ

**For Example:** What is the value of cos $20^o$ cos $40^o$ cos $60^o$ cos 80o?

**Solution:** We know cos θ * cos 2θ * cos 4θ = ¼ cos 3θ

Now, (cos $20^o$ cos $40^o$ cos $80^o$ ) cos $60^o$

¼ (Cos 3*20) * cos $60^o$

¼ $Cos^2$ $60^o$ = ¼ * $(½)^2$ = 1/16

# Trigonometry

**If** $a \sin \theta + b \cos \theta = m$ & $a \cos \theta - b \sin \theta = n$

then $a^2 + b^2 = m^2 + n^2$

## For Example:

If $4 \sin \theta + 3 \cos \theta = 2$ , then find the value of $4 \cos \theta - 3 \sin \theta$:

## Solution:

Let $2 \cos \theta - 3 \sin \theta = x$

By using formulae $a^2 + b^2 = m^2 + n^2$

$4^2 + 3^2 = 2^2 + x^2$

$16 + 9 = 4 + x^2$

$X = \sqrt{21}$

**If** $\sin \theta + \cos \theta = p$ & $\csc \theta - \sec \theta = q$

then $P - (1/p) = 2/q$

## For Example:

If $\sin \theta + \cos \theta = 2$ , then find the value of $\csc \theta - \sec \theta$:

# Trigonometry

**Solution:**

By using formulae:

P – (1/p) = 2/q

2-(1/2) = 3/2 = 2/q

Q = 4/3 or csc θ – sec θ = 4/3

## If

a cot θ + b csc θ = m & a csc θ + b cot θ = n

then $b^2 - a^2 = m^2 - n^2$

## If

cot θ + cos θ = x & cot θ – cos θ = y

then $x^2 - y^2 = 4\sqrt{xy}$

## If

tan θ + sin θ = x & tan θ – sin θ = y

then $x^2 - y^2 = 4\sqrt{xy}$

## If

$y = a^2 \sin^2 x + b^2 \csc^2 x + c$

# Trigonometry

$y = a^2 \cos^2 x + b^2 \sec^2 x + c$

$y = a^2 \tan^2 x + b^2 \cot^2 x + c$

## then,

ymin = 2ab + c

ymax = not defined

## For Example:

If $y = 9 \sin^2 x + 16 \csc^2 x + 4$ then ymin is:

## Solution:

For, y min = 2* √9 * √16 + 4

= 2*3*4 + 20 = 24 + 4 = 28

## If

$y = a \sin x + b \cos x + c$

$y = a \tan x + b \cot x + c$

$y = a \sec x + b \csc x + c$

**then,** ymin = + [$\sqrt{}$($a^2$+$b^2$)] + c

ymax = – [$\sqrt{}$($a^2$+$b^2$)] + c

## For Example:

If y = 1/(12sin x + 5 cos x +20) then ymax is:

## Solution:

For, y max = 1/x min

= 1/- ($\sqrt{}$$12^2$ +$5^2$) +20 = 1/(-13+20) = 1/7

**$Sin^2$ θ, maxima value = 1, minima value = 0**

**$Cos^2$ θ, maxima value = 1, minima value = 0**

## More form us:

# About Trignometric Identities Part-1

**Hello Readers,**

**Below in the post ,We shall discuss about Trignometric Identities of the Quant section. Now a days these topics have became an important part of the Quant test in SSC CGL, and other Govt Exam.,So Here We are providing short tricks and Quant quiz.**

# Trigonometry

(ii) $\sec^2\theta = \tan^2\theta = 1$

$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

If $\sec\theta \pm \tan\theta = x$ then, $\sec\theta \pm \tan\theta = 1/x$

(iii) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) = 1$

If $\operatorname{cosec}\theta \pm \cot\theta = x$ then $\operatorname{cosec}\theta \pm \cot\theta = \frac{1}{x}$

(iv) $\sin^2\theta + \cos^2\theta = 1$, $\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

(v) $\sec^2\theta - \tan^2\theta = 1$, $\sec^2\theta = 1 + \tan^2\theta$

$\tan^2\theta = \sec^2\theta - 1$.

(vi) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$, $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\cot^2\theta = \operatorname{cosec}^2\theta - 1$.

(v) If $\sin\theta + \cos\theta = a$ then, $\sin\theta - \cos\theta = \sqrt{2 - a^2}$

# Some questions based on above formulae :-

[1] The value of $(\tan 23^\circ \times \tan 67^\circ)$ is equal to :-

(a) 1 (b) 0 (c) 2 (d) None of these

[2] If $4\tan\theta = 3$, the value of $\left(\frac{4\sin\theta - 2\cos\theta}{4\sin\theta + 2\cos\theta}\right)$ is :-

(a) $1/5$ (b) $2/5$ (c) $5/3$ (d) None of these

[3] The value of $\left(\sin\frac{\pi}{6} + \cos\frac{\pi}{3} + \tan\frac{3\pi}{4}\right)$ is :-

(a) 1 (b) 0 (c) $-1$ (d) None of these.

[4] If $x \tan 45^\circ \cos 60^\circ = \sin 60^\circ \cdot \cot 60^\circ$, then $x$ is equal to :-

(a) 1 (b) $\sqrt{3}$ (c) $1/2$ (d) $\frac{1}{\sqrt{2}}$

[5] The value of $\sin 30^\circ \cos 60^\circ$ is :-

(a) $1 - \cos 30^\circ \sin 60^\circ$

(b) $1 - \cos 30^\circ \cos 60^\circ$

(c) $1 - \sin 30^\circ \sin 60^\circ$

(d) $1 - \sin 30^\circ \cos 60^\circ$

**Answers and Solution:**

# Trigonometry

(1)(a) $\tan 23^\circ \times \tan 67^\circ = 23^\circ \times \tan(90^\circ - 23)$

$= \tan 23^\circ \times \cot 23^\circ$

$= 1$

(2)(a) $4\tan\theta = 3 \Rightarrow \tan\theta = 3/4$

$\therefore \dfrac{4\sin\theta - 2\cos\theta}{4\sin\theta + 2\cos\theta} = \dfrac{4\frac{\sin\theta}{\cos\theta} - 2}{4\frac{\sin\theta}{\cos\theta} + 2} = \dfrac{4\tan\theta - 2}{4\tan\theta + 2}$

$\Rightarrow \dfrac{(4 \times \frac{3}{4} - 2)}{(4 \times \frac{3}{4} + 2)} = 1/5$

(3)(d) $\sin\frac{\pi}{6} + \cos\frac{\pi}{3} + \tan^3\frac{\pi}{4} = \sin 30^\circ + \cos 60^\circ + \tan^3 45^\circ$

$= \left(\frac{1}{2} + \frac{1}{2} + 1^3\right) = 2$

(4)(a) $x = \dfrac{\sin 60^\circ \cdot \cot 60^\circ}{\tan 45^\circ \cdot \cos 60^\circ} = \dfrac{(\sqrt{3}/2)(1/\sqrt{3})}{(1)(1/2)}$

$= \frac{1}{2} \times 2 = 1$

(5)(a) $\sin 30^\circ \cdot \cos 60^\circ = \frac{1}{2} \times \frac{1}{2} = 1/4$

Go through options

$\cos 30^\circ \cdot \sin 60^\circ = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} = 3/4$

$\therefore \sin 30^\circ \cos 60^\circ = 1 - \cos 30^\circ \sin 60^\circ$.

## About Trignometric Identities Part-II

**Hello Readers,**

**Below in the post ,We shall discuss about Trignometric Identities Part II of the Quant section. Now a days these topics have became an important part of the Quant test in SSC CGL, and other Govt Exam.,So Here We are providing short tricks and Quant quiz.**

# Trigonometry

## Important Trigonometrically Identities:

1. $sin^2\theta + cos^2\theta = 1$
2. $sin\theta = \sqrt{1 - cos^2\theta}$
3. $cos\theta = \sqrt{1 - sin^2\theta}$
4. $sec^2\theta + tan^2\theta = 1$
5. $sec^2\theta = 1 + tan^2\theta$
6. $tan^2\theta = sec^2\theta - 1$
7. $cos\,ec^2\theta - cot^2\theta = 1$
8. $cos\,ec^2\theta = 1 + cot^2\theta$
9. $cot^2\theta = cos\,ec^2\theta = 1$
10. $sin\theta\,.\,sos\,ec\,\theta = 1$
11. $\text{Sin}\theta = \frac{1}{cos\,ec\theta}$
12. $\text{Cosec}\theta = \frac{1}{sin\,\theta}$
13. $\text{Cos}.\,\text{sec}\theta = 1$
14. $\text{Cos}\,\theta = \frac{1}{sec\,\theta}$
15. $\text{sec}\theta = \frac{1}{cos\,\theta}$
16. $\tan\theta.\,cot\,\theta = 1$
17. $\tan\theta = \frac{1}{cot\,\theta}$
18. $\text{Cot}\,\theta = \frac{1}{tan\,\theta}$
19. $\tan\theta = \frac{sin\,\theta}{cos\,\theta}$
20. $\cot\theta = \frac{cos\,\theta}{sin\,\theta}$

**Some useful Results:**

1. If ax + by = m and ay – bx = n then $(a^2 + b^2)(x^2 + y^2) = m^2 + n^2$

   Where, x and y can be any trigonometric ratio i.e $sin\theta$, $cos\theta$, $tan\theta$ etc.
2. If a sin $\theta$ + b cos $\theta$ = c, and $c^2 = a^2 + b^2$ then

   Base = b, perpendicular = a and hypotenuse =

**Sum and Difference Formula: -**

1. sin (A+B) = sin A. cos B + cos A sin B
2. sin (A –B) = sin A. cos B – cos A sin B
3. cos (A+B) = cos A. cos B – sin A sin B
4. cos (A – B) = cos A. cos B + sin A sin B
5. 2 sin A. cos B = sin (A+B) + sin (A-B)
6. 2 cos A. sin B = sin (A +B) + sin (A-B)
7. 2 sin A. sin B = cos (A-B) – cos (A+B)
8. 2 cos A. cos B = cos (A +B) + cos (A-B)
9. $Sin^2 A - sin^2 B$ = sin (A+B). sin (A-B)
10. $Cos^2 A - cos^2 B$ = cos (A+B). cos (A-B)

# Trigonometry

## Some more Formula

1. $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$
2. $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$
3. $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$
4. $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$

## T – radius of Multiple Angles : -

1. $\sin 2A = 2\sin A.\cos A = \frac{2\tan A}{1+\tan^2 A}$
2. $\cos {}^2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$

   $1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$
3. $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$
4. $\sin 3A = 3\sin A - 4\sin 3A$
5. $\cos 3A = 4\cos 3A - 3\cos A$
6. $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

**Some Questions based on Above mention Formula.**

# Trigonometry

1. The value of $\sin^2 5° + \sin^2 25° + \sin^2 45° + \sin^2 65° + \sin^2 85°$ is equal to :

   (a) 1.5 (b) 2 (c) 2.5 (d) 3

2. If in $\Delta ABC, \angle A = 90°, BC = a, AC = b$ and $AB = c$, then the value of tan B + tan C is :

   (a) $\frac{c^2}{ab}$ (b) $\frac{a^2+c^2}{b}$ (c) $\frac{b^2}{ac}$ (d) $\frac{a^2}{bc}$

3. If A, B and C be the angles of a triangle, then which of the following is the incorrect relation is:

   (a) $\text{Sin}\frac{A+B}{2} = \cos\frac{C}{2}$

   (b) $\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$

   (c) $\tan\left(\frac{A+B}{2}\right) = \tan\frac{C}{2}$

   (d) $\cot\left(\frac{A+B}{2}\right) = \tan\frac{C}{2}$

4. The numerical value of $\cot 18° \left(\cot 72° \cot^2 22° + \frac{1}{\tan 72 \sec^2 68°}\right)$ is :

   (a) 1 (b) $\sqrt{2}$ (c) 3 (d) $1/\sqrt{3}$

5. If cosec 39 = x, the value of $\frac{1}{\text{cosec}^2 51°} + \sin^2 39° + \tan^2 51° - \frac{1}{\sin^2 51° \sec^2 39°}$ is :

   (a) $\sqrt{x^2-1}$ (b) $\sqrt{1-x^2}$

   (c) $x^2 - 1$ (d) $1 - x^2$

**Answers and solution**

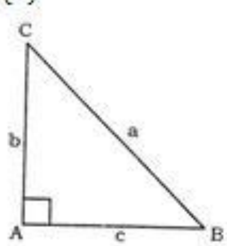
1. (C) $\text{Sin}^2 5° + \sin^2 85° + \sin^2 25° + \sin^2 65° + \sin^2 45°$

   $= \sin^2 5° + \sin^2 (90° = 5°) + \sin^2 25° + \sin^2 (90° - 25°) + \sin^2 45°$

   $= (\sin^2 5° + \cos^2 5°) + (\sin^2 25° + \cos^2 25°) + \sin^2 45°$

   $= 1 + 1 + ½ = 2 ½ = 2.5$

2. (d)




   tan B = AC/AB = b/c

   tan C = AB/AC = c/b

   tan B + tan c = b/c + c/b

   $= (b^2 + c^2)/bc = a^2/bc$

# Trigonometry

3. (c) $A + B + C = \pi$

$= A + B/2 = \pi/2 - c/2$

$= \sin (A+B)/2 = \sin (\pi/2 - c/2) = \cos C/2$

Similarly,

$\text{Cos } (A+B)/2 = \sin C/2$

$\text{Cot } (A+B)/2 = \cot c/2$

4. (a) $\text{Cot } 18° \left(\cot 72° . \cos^2 22° + \frac{1}{\tan 72° . \sec^2 68°}\right)$

$= \cot 18° (\cot 72° . \cos^2 22° + \cot 72° . \cos^2 68°)$

$= \cot 18°. \text{Cot } 72° (\cos^2 22° + \cos^2 68°)$

$= 1 * 1 = 1$

5. (c) $\frac{1}{\operatorname{cosec}^2 51°} + \sin^2 39° + \tan^2 51° - \frac{1}{\sin^2 51° \sec^2 39°}$

$= \sin^2 51° + \sin^2 39° + \tan^2 (90° - 39°) - \frac{1}{\sin^2 (90° - 39°) . \sec^2 39°}$

$= \cos^2 39° + \sin^2 39° + \cot^2 39° - \frac{1}{\cos^2 39° . \sec^2 39°}$

$[\because \sin(90° - \theta) = \cos\theta, \tan(90° - \theta) = \cot\theta]$

$= 1 + \cot^2 39° - 1 = \cot^2 39°$

$= \operatorname{cosec}^2 39° - 1 = x^2 - 1$