Quantitative Aptitude

Number system and number series

Definitions

Natural Number: These are the number (1, 2, 3 etc.) that are used for counting. In other words, all positive integers are natural number.

Prime Numbers: A natural number larger than unity is a prime number if it does not have other divisors except for itself and unity.

Some Properties of Prime Number

- The lowest prime number is 2.
- 2 is also only even prime number
- The lowest odd prime number is 3.
- The remainder when a prime number $p \geq 5$ is divided by 6 is 1 or 5. However, if a number on being divided by 6 give a remainder of 1 or 5 the number need not be prime
- The remainder of the division of the square of a prime number $p \geq 5$ divided by 24 is 1.
- For prime numbers $p > 3$, $p^2 - 1$ is divided by 24.
- Prime number between 1 to 100 are: 2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,91.
- Prime number between 100 to 200 are: 101,103,107,109,113,127,131,137,139,149,151,157,163,167,173,179,181,191,193,197,199.
- If *a* and *b* are any two odd primes then $a^2 – b^2$ is composite. Also, $a^2 + b^2$ is aomposite.
- The remainder of the division of the square of a prime number $p \geq 5$ divided by 12 is1.

To Check Whether a Number is Prime of Not

To check whether a number *N* is prime, adopt the following proses.

a) Take the square root of the number
b) Round of the square root to the immediately lowers integer. Call this number z. For example if you have to check for 181, its square root will be 3..... . Hence, the value of z, is this case will be 13.
c) Check for divisibility of the number *N* by all prime number below z. If there is no prime number below the value of z which divides *N* then the number *N* will be prime.

To illustrate:-

The value of √239 lies between 15 to 16. Hence , take the value of z as 16
Prime number less than 16 are 2, 3, 5, 7, 11 and 13, 239 is not divisible by any of these. Hence you can conclude that 239 is a prime number.

A Brief Look into Why this Works?

Suppose you are asked to find the factor of the number 40. An untrained mind will find the factor as : 1, 2, 4, 8, 10, 20 and 40.

The same task will be performed by a trained mind as follows:

1	X	40
2	X	20
4	X	10
5	X	8

and

i.e., The discovery of once factor will automatically yield the other factor. In other words, factor will appear in terms of what can be called as factor pairs. The locating of one factor, will automatically pinpoint the other one for you. Thus, in the example above, when you find 5 as a factor of 40 , you will automatically get 8 too as a factor.

Why do we not need to check with composite numbers below the square root?
This will gain be understood best if explained in the context of the example above. The only composite number in the list above is 9. You do not need to check with 9, because when you checked *N* for divisibility with 3 you would get either of two cases:

Case I: If N is divisible by 3: In such a case, *N* will automatically become non-prime and we can stop our checking. Hence you will not need to check for the divisibility of the number by 9.

Case II: N is not divisible by 3: If *N* not divisible by 3, it is obvious that it will not be disable by 9. Hence, you will not need to check for the divisibility of the number by 9.

Integers: A set which consists of natural numbers, negative integers (-1, -2, -3… -n…) and zero is known as the set of integers. The number belonging to this set are known as integers.

Composite Numbers: It is a natural number that has at least one divisor different from unity and itself.

Whole Numbers: The set of number that includes all natural numbers and the number zero are called whole numbers. Whole number are also called as Non-negative integers.

Real Numbers: all number that can be represented on the number line are called real numbers. Every real number can be approximately replaced with a terminating decimal.

Rational Numbers: A rational number is defined as number of the form *alb* where a and *b* are integers and b ≠ 0.

THE CONCEPT OF GCD (GREATEST COMMON DIVISOR OR HIGHEST COMMON FACTOR)

Consider two natural number n, and n_2. If number n_1 and n_2 are exactly divisible by the same number x, then x is a common divisor of n_1 and n_2. The highest of all the common divisor of n_1 and n_2 is called as the GCD or the HCF. This is denoted as GCD (n_1, n_2).

Rules for Finding the GCD of Two Number n_1 and n_2

a) Find the stander form from the number n_1 and $n_{2.}$
b) Write out all prime factor that are common to the standard forms of the numbers n_1 and n_2
c) Raise each of the common prime factor listed above to the lesser of the powers in which it appears in the standard forms of the number n_1 and n_2
d) The product of the results of the previous step will be the GCD of n_1 and $n_{2.}$

Example: find the GCD of 150, 210,375.

Step 1*:* Writing down the standard form of number

150 = 5 x 5 x 3 x 2

210 = 5 x 2 x 7 x 3

375 = 5 x 5 x 5 x 3

Step 2: Writing prime factors common to all the three numbers is $5^1\ 3^1$

Step 4: Hence, the HCF will be 5 x 3 = 15

For practice, find the HCF of the following:

(a) 78, 39, 195

(b) 440, 140, 390

(c) 198, 121, 1331

THE CONCEPT OF LCM (LEAST COMMON MULTIPLE)

Let $n_{1,}$ and n_2 natural numbers distinct from each other. The smallest natural number n that is exactly divisible by $n_{1,}$ and n_2 is called the Least Common Multiple (LCM) of $n_{1,}$ and n_2 and is designated as LCM ($n_{1,}$ n_2).

Rule for Finding the LCM of 2 numbers $n_{1,}$ and n_2

(a) Find the standard form of the numbers $n_{1,}$ and n_2
(b) Write out all the prime factors, which are contained in the standard forms of the numbers.

(c)Raise each of the prime factors listed above to the highest of the powers in which it appears in the appears in the standard forms of the numbers $n_{1,}$ and n_2

(d) the product of result of the previous step will be the LCM of $n_{1,}$ and n_2

Example: Find the LCM of 150, 210, 375.

***Step 1*:** Writing down the standard form of numbers

150 = 5 x 5 x 3 x 2

210 = 5 x 2 x 7 x 3

375 = 5 x 5 x 5 x 3

Step 2: Write down all the prime factor: that appear at least once in any of the numbers: 5, 3, 2, 7.

Step 3: Raise each of the prime factors to their highest available power (considering each to the numbers).

The LCM = $2^1 \times 3^1 \times 5^3 \times 7^1$ = 5250.

Important Rule:

GCD $(n_{1,}\ n_2)$.LCM $(n_{1,}\ n_2) = n_{1,}\ n_2$

i.e. The product of the HCF and the LCM equals the product of the numbers.

RULE FOR FINDING OUT HCF AND LCM OF FRACTIONS:

(A) HCF of two of more fractions is given by:

HCF of Numerators/LCM of Denominators

(B) LCM of two or more fractions is given by:

LCM of Numerators/HCF Denominators

Rules for HCF: If the HCF of x and y is G, then the HCF of

(i) x, (x + y) is also G
(ii) x, (x – y) is also G
(iii) (x + y), (x – y) is also G