Probability and Counting Techniques Tips

I. Counting techniques:

A. General:

1. Sometimes the number of outcomes of your sample space is too large to list and count. So counting techniques come in handy.
a. For example there are over 2.5 million five card poker hands.

B. Multiplication rule:

1. If the first event is done in x ways, and then a second event is done in y ways, then the first event followed by the second event has xy possible ways in which the events can be done.

a. **For example-** There are five different roads that lead from your house to your significant others house, and then four different roads that lead to the movie theater. How many different routes can you take to get to the movies? All you gotta do is take the number of possible paths and multiply them. 5 X 4 = 20 possible paths from home to the movies.

C. Combinations vs. permutations:

1. **Permutation**- is an ordered arrangement of **k** items from a larger group of **n** possible choices,

a. Formula- n!/(n-k)!

b. **For example-**

The abp decides to assign you a password consisting of 5
letters. You are learning statistics, so you want to see if you
learned anything. So you decide to see if you can calculate
the number of possible passwords:
It is definitely a permutation because the order of the letters in your password is important. There are 26 letters in the alphabet, and your password is 5 letters long so:

(26/5)= 26!/(26-5)!= = 7893600 possible passwords!

2. What if the person wanted to make the password 26 letters long(crazy, huh, but humor me for the sake of the concept).
a. The denominator would simplify into 0! (Which equals 1 by
b. This results in **n!** being the formula if you want to see the number of possible n possible permutations. **So 26! = over 4 e 26 possible outcomes.**

a. Formula- **n!/k!(n-k)!**

b. For example- A committee of 4 must be selected out of 10 people to choose from. How many different combinations can you obtain?

(10/4) = 10!/4!(10-4)! = 210

*Note- there always should always be more permutations than combinations.

II. General- A. Equally likely to occur:

Probability

= number of desired events/the total number of probable outcomes (the number of events in your sample space)

1. for example-

a. The probability of flipping a coin is ½. This is due to the fact that a coin can only land on one side, and there are two possible sides to choose from.
b. The probability of pulling a specific card from a deck is 1/52 because you are pulling one card out of 52 choices.

***Note- the value of your probability should always be between 0 and 1.**

III. Conditional probability:

A. General:
1. a. If the probability of something hinges on something else, or depends on some condition or premise, then the following conditional probability formula must be used:

$$P(A|B)= p(A\cap B)/P(B)$$

b. For example

Suppose that you roll a pair of dice. If the sum of the two die is 4, what is the probability that the each die had a 2?

Event A is rolling two 2's, and B is the sum of the two die is 4.

The sample space is : { (1,3);(2,2);(3,1)}

$$P(A|B)= p(A\cap B)/P(B) = 1/13$$

2. Sometimes, if you know the conditional probability, the formula

can be manipulated to find P (A∩B).

$$P(A\cap B)= P (B) X P (A|B)$$

For example,

I open two companies that split up my corporation's workload. My corporation is making statistical calculators. The load is divided up so that branch A has 40 percent of the workload. If the company A has 2 percent of the calculators come up defective, and company B has 3 percent defective, what are the probabilities of defective calculators from A? From B?

To solve this problem easily, the use of a tree diagram is required. see nxt page!

IV. Addition vs. Multiplication:

A. General:

1. **Mutually exclusive-** if two events have nothing in common or they cannot occur simultaneously. For instance, you cannot roll a 5 and a 6 on a die, or land on heads and tails.

2. Addition- When looking for the union of mutually exclusive events, the addition of the individual probabilities is required.

a. For example,

What is the probability of pulling an ace out a deck of cards? There are four aces, one for each suit, and each has a probability of 1/52. P(ace)= P(ace of spaids) + P(ace of diamonds) + P(ace of clubs) + P(ace of hearts)= 1/52 + 1/52 + 1/52 + 1/52= 4/52 = 1/13 Or if you wanna use the general probability formula in the section above there are 4 aces out of 52 cards. Therefore, P(ace) = 4/52 = 1/13.

3. **Independent events**- events that have no effect on each other. The probability with or without the condition is the same. Given that P(A) and P(B) are both > 0, then:
a. $P(B|A) = P(B)$
b. $P(A|B) = P(A)$
c. $P(A\cap B) = P(A)P(B$
d. For example, the probability that you are reading this handout given that a foreign immigrant is applying for citizenship in Oregon should be the same as the probability that you are reading this handout. The probability that a thunderstorm hits your city should given that you overslept this morning is the same as the probability of the thunderstorm hitting your city. The fact that you overslept has nothing to do with the weather.

4. Multiplication- If two events are independent, the probability of them occurring together (intersection-∩ is the same as the product of the individual probabilities. (Rule 3c above)

a. The probability of getting two heads when flipping a fair coin is equal to the product of each flip since the outcome of the first doesn't affect the outcome of the second flip. So, $P(H\cap H) = P(H)P(H) = (½)(½) = ¼$

5. Replacement: The probability of pulling two aces out of a deck(given that you put back the first one after pulling it) would be:

$P(A \cap A) = 4/52 \times 4/52 = 1/169$

6. Without Replacement: If you didn't replace the ace, the problem would change slightly due to the fact that there is one less ace and card in the deck:

$P(A \cap A) = 4/52 \times 3/51 = 0.45\%$