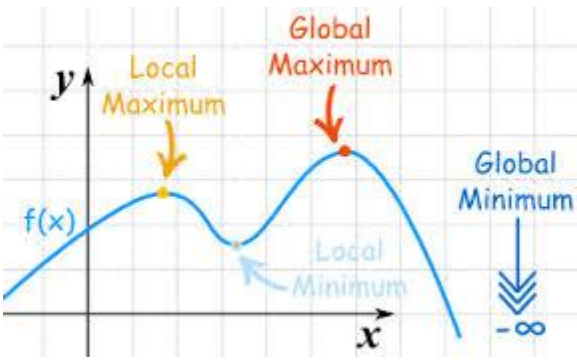
Limit and Derivatives

The concepts of straight line, maxima and minima, global maxima and minima, Rolle's Theorem and LMVT all come under the head of Application of Derivatives.

- If a function is increasing on some interval then the slope of the tangent is positive at every point of that interval due to which its derivative is positive.
- Similarly, the derivative of a function which is decreasing on some interval is negative as the slope of the tangent is negative at every point of that interval.



- A function f is said to have a local maximum (also termed as relative maximum) at x = a if f(x) ≤ f(c), for every x in some open interval around x = c.
- A function f is said to have a relative minimum or a local minimum around x = c if f(x) ≥ f(c), for every x in some open interval around x = a.
- A function f is said to have a global maximum (also termed as absolute maximum) at x = a if f(x) ≤ f(c), for every x in the domain under consideration.
- A function f is said to have an absolute minimum or a global minimum around x = c if f(x) ≥ f(c), for every x in the whole domain under consideration.

Rolle's Theorem

Let y = f (x) be a given function which satisfies the conditions:

1) f (x) is continuous in [a , b]

2) f (x) is differentiable in (a , b)

3) f (a) = f (b)

Then f'(x) = 0 at least once for some x ∈ (a, b).

- Certain points to be noted in Rolle'sTheorem include:

- Converse of the theorem does not hold good.
- There can be more than one such c.
- The conditions of Rolle's Theorem are only sufficient and not necessary.

Lagrange Mean Value Theorem (LMVT)

- If a given function y = f (x) satisfies certain conditions like:

f(x) is continuous in [a , b]

f(x) is differential in (a, b)

then f'(x) = [f(b) – f(a)]/[b–a] for some x ∈ (a, b). This is the generalization of the Rolle's Theorem and is termed as Lagrange Mean Value theorem.

- A function is said to be monotonically increasing at x = a if f(x) satisfies f(a+h) > f(a) and f(a-h) < f(a), for some small positive h.
- A function is said to be monotonically decreasing at x = a if f(x) satisfies f(a+h) < f(a) and f(a-h) > f(a), for some small positive h.
- If f'(x) > 0 ∀ x ∈ (a,b) and points which make equal to zero (in between (a, b)) don't form an interval, then f (x) would be increasing in [a, b] otherwise it will be non-decreasing function.
- If f'(x) > 0 ∀ x ∈ (a,b) and points which make equal to zero (in between (a, b)) don't form an interval, f (x) would be decreasing in [a, b], otherwise it will be non-increasing.
- For all x and y, such that x≤ y, if f(x) ≤ f(y), then the function f is said to be monotonically increasing, increasing or non-decreasing.
- Similarly, for x ≤ y, if f(x) ≥ f(y), then the function is monotonically decreasing, decreasing or non-increasing i.e. it reverses the order.
- If f is increasing for x > a and f is also increasing for x < a then f is also increasing at x = a provided f(x) is continuous at x = a.
- If f(x) is strictly increasing, then f^{-1} exists and is also strictly increasing.
- If f(x) is strictly increasing on [a, b] and is also continuous then f^{-1} is continuous on [f(a), f(b)].
- If f(x) and g(x) are strictly increasing (decreasing) functions on [a, b], then gof(x) is strictly increasing (decreasing) function on [a, b].
- If one of the two functions f(x) and g(x) is strictly increasing and other is strictly decreasing then gof(x) is strictly decreasing on [a, b].

Limit and Derivatives

- If a continuous function y = f(x) is strictly increasing in the closed interval [a, b], then f(a) is the least value.
- If f(x) is decreasing in [a, b], then f(b) is the least and f(a) is the greatest value of f(x) in [a, b].
- If f(x) is non-monotonic in [a, b] and is continuous then the greatest and the least value of f(x) in [a, b] are those where f(x) = 0 or f'(x) does not exist or at the extreme values.
- The direction of acceleration is in the direction of velocity or opposite to it.
- When the particle is going upward, the value of g is negative and when it is coming back, the value of g is positive.
- At maximum height the velocity of a particle is zero. The value of g is 9.8 m/s^2 or 980 cm/s^2.
- Slope of tangent to the curve y = f(x) at the point (x, y) is m = tan θ = $[dy/dx]_{(x,y)}$
- If the equation of the curve is in the parametric form x = f(t) and y = g(t), then the equations of the tangent and the normal are y – g(t) = g'(t)/f'(t)(x–f(t)) and f'(t)[x–f(t)] + g'(t) [y–g(t)] = 0 respectively.
- The equation of tangent to the curve y = f(x) at the point $P(x_1, y_1)$ is given by $y - y_1 = [dy/dx]_{(x,y)} (x - x_1)$
- If dy/dx = 0 then the tangent to curve y = f(x) at the point (x, y) is parallel to the x-axis.
- If dy/dx → ∞, dx/dy = 0, then the tangent to the curve y = f(x) at the point (x, y) is parallel to the y-axis.
- If dy/dx = tan θ > 0, then the tangent to the curve y = f(x) at the point (x, y) makes an acute angle with positive x-axis and vice versa.
- If two curves are orthogonal, then the product of their slopes is -1 everywhere wherever they intersect.
- **Length of tangent, normal, subtangent, subnormal:**

Tangent = $\left|\dfrac{y\sqrt{1+(dy/dx)^2}}{dy/dx}\right|$

Subtangent = $\left|\dfrac{y}{dy/dx}\right|$

Normal = $\left|y\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\right|$

Subnormal = $\left|y\left(\frac{dy}{dx}\right)\right|$