Area Of Plane Figures

EXPECTED BACKGROUND KNOWLEDGE

• Simple closed figures like triangles, quadrilaterals, parallelograms, trapeziums, squares, rectangles, circles and their properties.

• Different units for perimeter and area such as m and m2 , cm and cm2 , mm and mm2 and so on.

• Conversion of one unit into other units.

• Bigger units for areas such as acres and hectares.

• Following formulae for perimeters and areas of varioius figures:

I. Perimeter of a rectangle = 2 (length + breadth)
II. Area of a rectangle = length × breadth
III. Perimeter of a square = 4 × side
IV. Area of a square = (side)2
V. Area of a parallelogram = base × corresponding altitude
VI. Area of a triangle = $\frac{1}{2}$ base × corresponding altitude
VII. Area of a rhombus = $\frac{1}{2}$ product of its diagonals
VIII. Area of a trapezium = $\frac{1}{2}$ (sum of the two parallel sides) × distance between them
IX. circumference of a circle = 2 π × radius (x) Area of a circle = π× $(\text{radius})^2$

20.1 PERIMETERS AND AREAS OF SOME SPECIFIC QUADRILATEALS AND TRIANGLES

You already know that the distance covered to walk along a plane closed figure (boundary) is called its perimeter and the measure of the region enclosed by the figure is called its area. You also know that perimeter or length is measured in linear units, while area is measured in square units. For example, units for perimeter (or length) are m or cm or mm and that for area are m^2 or cm^2 or mm^2 (also written as sq.m or sq.cm or sq.mm).

You are also familiar with the calculations of the perimeters and areas of some specific quadrilaterals (such as squares, rectangles, parallelograms, etc.) and triangles, using certain formulae. Lets us consolidate this knowledge through some examples.

Area Of Plane Figures

Example 20.1: Find the area of square whose perimeter is 80 m.

Solution: Let the side of the square be a m.

So, perimeter of the square = 4 × a m

Therefore, 4a = 80

or a $= \frac{80}{4} = 20$

That is, side of the square = 20 m

Therefore, area of the square = $(20m)^2 = 400\ m^2$

Example 20.2: Length and breadth of a rectangular field are 23.7 m and 14.5 m respectively.

Find:

(i) barbed wire required to fence the field

(ii) area of the field.

Solution: (i) Barbed wire for fencing the field = perimeter of the field

= 2 (length + breadth)

= 2(23.7 + 14.5) m = 76.4 m

(ii) Area of the field = length × breadth

= 23.7 × 14.5 m2

= 343.65 m^2

Example 20.3: Find the area of a parallelogram of base 12 cm and corresponding altitude 8 cm.

Solution: Area of the parallelogram = base × corresponding altitude

= 12 × 8 cm^2

= 96 cm^2

Example 20.4: The base of a triangular field is three times its corresponding altitude. If the cost of ploughing the field at the rate of ` 15 per square metre is ` 20250, find the base and the corresponding altitutde of the field.

Solution: Let the corresponding altitude be x m.

Therefore, base = 3x m

So, area of the field = $\frac{1}{2}$ base × corresponding altitude

= $\frac{1}{2}$ 3x × x m2 = $3x^2/m^2$ .... (1)

Also, cost of ploughing the field at ` 15 per m2 = ` 20250

Therefore, area of the field = $\frac{20250}{15}$ m^2

= 1350 m^2 ... (2)

From (1) and (2), we have:

$3x^2/2 = 1350$

Or $x^2 = \frac{1350 \times 2}{3} = 900 = (30)^2$

Or x = 30

Hence, corresponding altitutde is 30 m and the base is 3 × 30 m i.e., 90 m.

Example 20.5: Find the area of a rhombus whose diagonals are of lengths 16 cm and 12 cm.

Solution: Area of the rhombus = $\frac{1}{2}$ product of its diagonals = $\frac{1}{2}$ × 16 × 12 cm^2

Example 20.6: Length of the two parallel sides of a trapezium are 20 cm and 12 cm and the distance between them is 5 cm. Find the area of the trapezium.

Solution: Area of a trapezium = $\frac{1}{2}$ (sum of the two parallel sides)×distance between them

= $\frac{1}{2}$ (20 + 12) × 5 cm^2 = 80 cm^2

CHECK YOUR PROGRESS 20.1

1. Area of a square field is 225 m^2. Find the perimeter of the field.

2. Find the diagonal of a square whose perimeter is 60 cm. 3. Length and breadth of a rectangular field are 22.5 m and 12.5 m respectively. Find:

I. Area of the field
II. Length of the barbed wire required to fence the field

4. The length and breadth of rectangle are in the ratio 3 : 2. If the area of the rectangle is 726 m^2, find its perimeter.

5. Find the area of a parallelogram whose base and corresponding altitude are respectively 20 cm and 12 cm.

6. Area of a triangle is 280 cm^2. If base of the triangle is 70 cm, find its corresponding altitude.

7. Find the area of a trapezium, the distance between whose parallel sides of lengths 26 cm and 12 cm is 10 cm.

8. Perimeter of a rhombus is 146 cm and the length of one of its diagonals is 48 cm. Find the length of its other diagonal.

20.2 HERON'S FORMULA

If the base and corresponding altitude of a triangle are known, you have already used the formula:

Area of a triangle = $\frac{1}{2}$ base × corresponding altitude

However, sometimes we are not given the altitude (height) corresponding to the given base of a triangle. Instead of that we are given the three sides of the triangle. In this case also, we can find the height (or altitude) corresponding to a side and calculate its area. Let us explain it through an example

Example 20.7: Find the area of the triangle ABC, whose sides AB, BC and CA are respectively 5 cm, 6 cm and 7 cm.

Solution: Draw AD ⊥ BC as shown in Fig. 20.1.

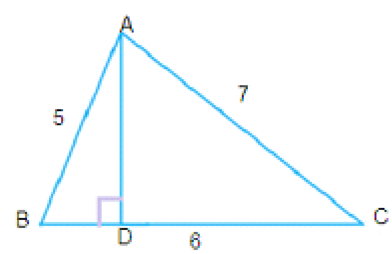

Fig. 20.1

Let BD = x cm

So, CD = (6 – x) cm

Now, from right triangle ABD, we have:

$AB^2 = BD^2 + AD^2$ (Pythagoras Theorem)

i.e. $25 = x^2 + AD^2$ ... (1)

Similarly, from right triangle ACD, we have

$AC^2 = CD^2 + AD^2$

i.e. $49 = (6 – x)^2 + AD2$ ... (2)

From (1) and (2), we have:

$49 – 25 = (6 – x)^2 – x^2$

i.e. $24 = 36 - 12x + x^2 - x^2$

or $12x = 12$, i.e., $x = 1$

Putting this value of x in (1), we have:

$25 = 1 + AD^2$

i.e. $AD^2 = 24$ or $AD = \sqrt{24} = 2\sqrt{6}$ cm

Thus, area of ΔABC = $\frac{1}{2}$ BC × AD = $\frac{1}{2} \times 6 \times 2\sqrt{6}$ cm^2 = $6\sqrt{6}$ cm^2

You must have observed that the process involved in the solution of the above example is lengthy. To help us in this matter, a formula for finding the area of a triangle with three given sides was provided by a Greek mathematician Heron (75 B.C. to 10 B.C.). It is as follows:

Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

where, a, b and c are the three sides of the triangle and $s = \frac{a+b+c}{2}$ This formula can be proved on similar lines as in Example 20.7 by taking a, b and c for 6, 7 and 5 respectively. Let us find the area of the triangle of Example 20.7 using this formula.

Here, a = 6 cm, b = 7 cm and c = 5 cm

So, $s = \frac{6+7+5}{2}$ = 9 cm

Therefore, area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{9\ (9 - 6)\ (9 - 7)\ (9 - 5)}$ 9−6 9−7 9−5 cm^2

= $\sqrt{9\times3\times 2\times3}$ cm^2

= $6\sqrt{6}$ cm^2, which is the same as obtained earlier

Let us take some more examples to illustrate the use of this formula.

Example 20.8: The sides of a triangular field are 165 m, 154 m and 143 m. Find the area of the field.

Solution: $s = \frac{a+b+c}{2} = \frac{(165+154+143)}{2}$ m 231m

So, area of the field = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{231\times (231 - 165)\ (231-154)\ (231-143)}$ m^2

= $\sqrt{231\times66\times77\times88}$ m^2

= $\sqrt{11\times3\times7\times11\times 2\times3\times11\times7\times11\times 2\times 2\times 2}$ m^2

$= 11 \times 11 \times 3 \times 7 \times 2 \times 2$ m2 = 10164 m^2

Example 20.9: Find the area of a trapezium whose parallel sides are of lengths 11 cm amd 25 cm and whose non-parallel sides are of lengths 15 cm and 13 cm.

Solution: Let ABCD be the trapezium in which AB = 11 cm, CD = 25 cm, AD = 15 cm and BC =13 cm (See Fig. 20.2)

Through B, we draw a line parallel to AD to intersect DC at E. Draw BF $\perp$ DC

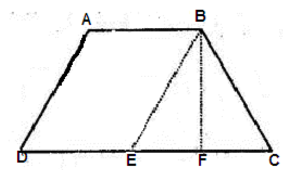

Fig. 20.2

Now, clearly BE = AD = 15 cm

BC = 13 cm (given)

and EC = (25 – 11) cm = 14 cm

So, for ΔBEC, s $= \frac{15+13+14}{2}$ cm = 21 cm

Therefore area of ΔBEC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21 \times (21-15)(21-13)(21-14)}$ cm^2

$= \sqrt{21 \times 6 \times 8 \times 7}$ cm^2

$= 7 \times 3 \times 4$ cm^2 = 84 cm^2 ... (1)

Again, area of ΔBEC = $\frac{1}{2}$ EC × BF

$= \frac{1}{2} \times 14 \times$ BF ... (2)

So, from (1) and (2), we have:

$\frac{1}{2} \times 14 \times$ BF = 84

i.e., BF = $\frac{84}{7}$ cm = 12 cm

Therefore, area of trapezium ABCD = $\frac{1}{2}$ (AB + CD) × BF

$= \frac{1}{2}(11 + 25) \times 12$ cm^2

$= 18 \times 12$ cm2 = 216 cm^2

Area Of Plane Figures

CHECK YOUR PROGRESS 20.2

1. Find the area of a triangle of sides 15 cm, 16 cm and 17 cm.

2. Using Heron's formula, find the area of an equilateral triangle whose side is 12 cm. Hence, find the altitude of the triangle.

20.3 AREAS OF RECTANGULAR PATHS AND SOME RECTILINEAR FIGURES

You might have seen different types of rectangular paths in the parks of your locality. You might have also seen that sometimes lands or fields are not in the shape of a single figure. In fact, they can be considered in the form of a shape made up of a number of polygons such as rectangles, squares, triangles, etc. We shall explain the calculation of areas of such figures through some examples.

Example 20.10: A rectangular park of length 30 m and breadth 24 m is surrounded by a 4 m wide path. Find the area of the path.

Solution: Let ABCD be the park and shaded portion is the path surrounding it (See Fig. 20.3).

So, length of rectangle EFGH = (30 + 4 + 4) m = 38 m

and breadth of rectangle EFGH = (24 + 4 + 4) m = 32 m

Therefore, area of the path = area of rectangle EFGH – area of rectangle ABCD

$= (38 \times 32 - 30 \times 24)\ m^2$

$= (1216 - 720)\ m^2$

$= 496\ m^2$

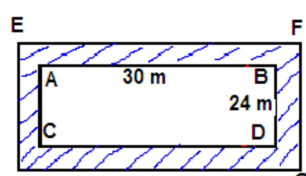

Fig. 20.3

Example 20.11: There are two rectangular paths in the middle of a park as shown in Fig. 20.4. Find the cost of paving the paths with concrete at the rate of ` 15 per m2 . It is given that AB = CD = 50 m, AD = BC = 40 m and EF = PQ = 2.5 m.

Solution: Area of the paths = Area of PQRS + Area of EFGH – area of square MLNO

$= (40 \times 2.5 + 50 \times 2.5 - 2.5 \times 2.5)\ m^2$

$= 218.75\ m^2$

So, cost of paving the concrete at the rate of ` 15 per m2 = 218.75 × 15

= 3281.25

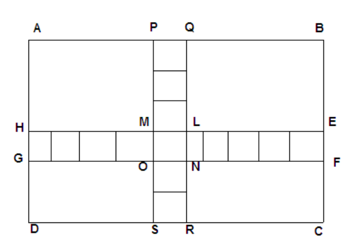

Fig. 20.4

Area Of Plane Figures

LET US SUM UP

• Perimeter of a rectangle = 2 (length + breadth)

• Area of a rectangle = length × breadth

• Perimeter of a square = 4 × side

• Area of a square = (side)2

• Area of a parallelogram = base × corresponding altitude

• Area of a triangle = $\frac{1}{2}$ base × corresponding altitude and also √s(s −a) (s −b)(s −c) where a, b and c are the three sides of the triangle and s = $\frac{a+b+c}{2}$

• Area of a rhombus = $\frac{1}{2}$ product of its diagonals

• Area of a trapezium = $\frac{1}{2}$ (sum of the two parallel sides) × distance between them

• Area of rectangular path = area of the outer rectangle – area of inner rectangle

• Area of cross paths in the middle = Sum of the areas of the two paths – area of the common portion

• circumference of a circle of radius r = 2 πr

• Area of a circle of radius r = πr 2

• Area of a circular path = Area of the outer circle – area of the inner circle

• Length l of the arc of a sector of a circle of radius r with central angle θ is $l = \pi r\theta/180^0$

• Perimeter of the sector a circle with radius r and central angle θ = $2r + \pi r\theta/180^0$

• Area of the sector of a circle with radius r and central and θ = $\pi r^2\theta/180^0$

• Areas of many rectilinear figures can be found by dividing them into known figures such as squares, rectangles, triangles and so on.

• Areas of various combinations of figures and designs involving circles can also be found by using different known formulas.

ss

Area Of Plane Figures