Permutation and Combination

- The concept of permutation is used for the arrangement of objects in a specific order i.e. whenever the order is important, permutation is used.

- The total number of permutations on a set of n objects is given by n! and is denoted as ${}^{n}P_{n} = n!$
- The total number of permutations on a set of n objects taken r at a time is given by ${}^{n}P_{r} = n!/ (n-r)!$
- The number of ways of arranging n objects of which r are the same is given by n!/ r!
- If we wish to arrange a total of n objects, out of which 'p' are of one type, q of second type are alike, and r of a third kind are same, then such a computation is done as n!/p!q!r!
- Al most all permutation questions involve putting things in order from a line where the order matters. For example ABC is a different permutation to ACB.
- The number of permutations of n distinct objects when a particular object is not to be considered in the arrangement is given by ${}^{n-1}P_{r}$
- The number of permutations of n distinct objects when a specific object is to be always included in the arrangement is given by $r.{}^{n-1}P_{r-1}$.
- If we need to compute the number of permutations of n different objects, out of which r have to be selected and each object has the probability of occurring once, twice or thrice… up to r times in any arrangement is given by $(n)^{r}$.
- Circular permutation is used when some arrangement is to be made in the form of a ring or circle.
- When 'n' different or unlike objects are to be arranged in a ring in such a way that the clockwise and anticlockwise arrangements are different, then the number of such arrangements is...
- If certain objects are to be arranged in such a way that the order of objects is not important, then the concept of combinations is used.
- The number of combinations of n things taken r ($0 \leq r \leq n$) at a time is given by ${}^{n}C_{r} = n!/r!(n-r)!$
- The relationship between combinations and permutations is ${}^{n}C_{r} = {}^{n}P_{r}/r!$
- The number of ways of selecting r objects from n different objects subject to certain condition like:

    1. k particular objects are always included = ${}^{n-k}C_{r-k}$

2. k particular objects are never included = ${}^{n-k}C_r$

- The number of arrangement of n distinct objects taken r at a time so that k particular objects are

    1. Always included = ${}^{n-k}C_{r-k}.r!$,
    2. Never included = ${}^{n-k}C_r.r!$.

- In order to compute the combination of n distinct items taken r at a time wherein, the chances of occurrence of any item are not fixed and may be one, twice, thrice, …. up to r times is given by ${}^{n+r-1}C_r$
- If there are m men and n women (m > n) and they have to be seated or accommodated in a row in such a way that no two women sit together then total no. of such arrangements

    = ${}^{m+1}C_n$. m! This is also termed as the Gap Method.

- If there is a problem that requires n number of persons to be accommodated in such a way that a fixed number say 'p' are always together, then that particular set of p persons should be treated as one person. Hence, the total number of people in such a case becomes (n-m+1). Therefore, the total number of possible arrangements is (n-m+1)! m! This is also termed as the String Method.
- Let there be n types of objects with each type containing at least r objects. Then the number of ways of arranging r objects in a row is n^r.
- The number of selections from n different objects, taking at least one

    = $\mathbf{{}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n = 2^n - 1.}$

- Total number of selections of zero or more objects from n identical objects is n+1

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- **Selection when both identical and distinct objects are present:**
- The number of selections, taking at least one out of $a_1 + a_2 + a_3 + ... a_n + k$ objects, where a_1 are alike (of one kind), a_2 are alike (of second kind) and so on ... a_n are alike (of nth kind), and k are distinct

    = $\{[(a_1 + 1)(a_2 + 1)(a_3 + 1) ... (a_n + 1)]2^k\} - 1$.

- Combination of n different things taken some or all of n things at a time is given by $2^n - 1$.

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- Combination of n things taken some or all at a time when p of the things are alike of one kind, q of the things are alike and of another kind and r of the things are alike of a third kind

  $= [(p + 1)(q + 1)(r + 1)\ldots.] - 1$

- Combination of selecting s_1 things from a set of n_1 objects and s_2 things from a set of n_2 objects where combination of s_1 things and s_2 things are independent is given by...