Friction in solids and liquids

ELASTICITY In solids, the atoms and molecules are free to vibrate about their mean positions. If this vibration increases sufficiently, molecules will shake apart and start vibrating in random directions. At this stage, the shape of the material is no longer fixed, but takes the shape of its container. This is liquid state. Due to increase in their energy, if the molecules vibrate at even greater rates, they may break away from one another and assume gaseous state. Water is the best example for this changing of states. Ice is the solid form of water. With increase in temperature, ice melts into water due to increase in molecular vibration. If water is heated, a stage is reached where continued molecular vibration results in a separation among the water molecules and therefore steam is produced. Further continued heating causes the molecules to break into atoms.

Elasticity

When an external force is applied on a body, which is not free to move, there will be a relative displacement of the particles. Due to the property of elasticity, the particles tend to regain their original position. The external forces may produce change in length, volume and shape of the body. This external force which produces these changes in the body is called deforming force. A body which experiences such a force is called deformed body. When the deforming force is removed, the body regains its original state due to the force developed within the body. This force is called restoring force. The property of a material to regain its original state when the deforming force is removed is called elasticity. The bodies which possess this property are called elastic bodies. Bodies which do not exhibit the property of elasticity are called plastic. The study of mechanical properties helps us to select the material for specific purposes. For example, springs are made of steel because steel is highly elastic

Stress and strain

In a deformed body, restoring force is set up within the body which tends to bring the body back to the normal position. The magnitude of these restoring force depends upon the deformation caused. This restoring force per unit area of a deformed body is known as stress. This is measured by the magnitude of the deforming force acting per unit area of the body when equilibrium is established.

$$Stree = restoring\ force/\ Area$$

Unit of stress in S.I. system is N/m2 . When the stress is normal to the surface, it is called Normal Stress. The normal stress produces a achange in length or a change in volume of the body. The normal stress to a wire or a body may be compressive or tensile (expansive) according as it produces a decrease or increase in length of a wire or volume of the body. When the stress is tangential to the surface, it is called tangential (shearing) stress

Solved Numerical

Q) A rectangular bar having a cross-sectional area of 28 mm2 has a tensile force of a 7KN applied to it. Determine the stress in the bar

Solution

Cross-sectional area A = $25mm^2 = 28\times(10^{-3})^2 = 28\times 10^{-6}\ m^2$ Tensile force F = 7KN = 7×10^3N

$$Stree = 7 \times 10^3 / 28 \times 10^{-6} = 0.25 \times 10^9 N/m^2$$

Strain

The external force acting on a body cause a relative displacement of its various parts. A change in length volume or shape takes place. The body is then said to be strained. The relative change produced in the body under a system of force is called strain

$$Strain\ (\varepsilon) = Change\ in\ dimension\ /original\ dimension$$

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Strain has no dimensions as it is a pure number. The change in length per unit length is called linear strain. The change in volume per unit volume is called Volume stain. If there is a change in shape the strain is called shearing strain. This is measured by the angle through which a line originally normal to the fixed surface is turned

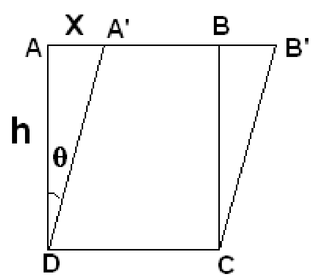
Longitudinal Strain: The ratio of change in length to original length

$$\varepsilon_l = \Delta l / l$$

Volume strain

$$\varepsilon_v = \Delta v / v$$

Shearing strain

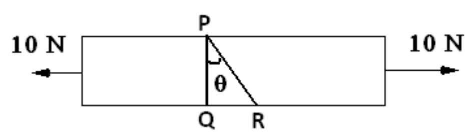


In figure a body with square cross section is shown a tangential force acts on the top surface AB causes shift of Surface by ‘X’ units shown as surface A’B’ ,thus side DA’ now mates an angle of θ with original side DA of height h

$$\varepsilon_S = x/\ h = tan\theta$$

Solved Numerical

Q) As shown in figure 10N force is applied at two ends of a rod. Calculate tensile stress and shearing stress for section PR. Area of cross-section PQ is 10 cm^2 , θ=30^O



Solution

Given cross-section area of PQ = 10 cm^2
Now PQ = PRcosθ
10 = PRcos30
10= PR (√3 /2) PR = 20/√3 cm^2 or 2/√3 m^2
Now normal force to area PR will be Fcos30 = 10 ×(√3 /2) = 5√3 N
Tangential force to area PR will be Fsin30 = 10×(1/2) = 5 N
∴ Tensile stress for section PR

$$\sigma_l = normal\ force\ /\ area\ of\ PR = 5\sqrt{3} / 2/\sqrt{3} \times 10^{-3} = 7.5 \times 10^3\ N/m^2$$

Shearing stress for section PR

$$\sigma_t = tangential\ force\ /\ area\ of\ PR = 5 / 2/\sqrt{3} \times 10^{-3} = 2.5\sqrt{3} \times 10^3\ N/m^2$$

Hooke’s Law and types of moduli
According to Hooke’s law, within the elastic limit, strain produced in a body is directly proportional to the stress that produces it.

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$$stress/strain = constant = \lambda$$

Where λ is called modulus of elasticity. Its unit is N m^{-2} and its dimensional formula is $ML^{-1}T^{-2}$. Depending upon different types of strain, the following three moduli of elasticity are possible

(i) Young's modulus: When a wire or rod is stretched by a longitudinal force the ratio of the longitudinal stress to the longitudinal strain within the elastic limits is called Young's modulus

$$Young\ 's modulus\ (Y) = Longitudinal\ stress\ /\ linear\ strain$$

Consider a wire or rod of length L and radius r under the action of a stretching force applied normal to its face. Suppose the wire suffers a change in length l then

$$Longitudinal\ stress = F/\pi r^2$$

$$Linear\ strain = l/L$$

$$Young\ 's modulus\ (Y) = F/\pi r^2 / l/L = FL/\pi r^2 l$$

(ii) Bulk modulus: When a solid or fluid (liquid or gas) is subjected to a uniform pressure all over the surface, the shape remains the same, but there is a change in volume. The force perunit area applied normally and uniformly over the surface is called normal stress. The change in volume per unit volume is called volume or bulk strain.

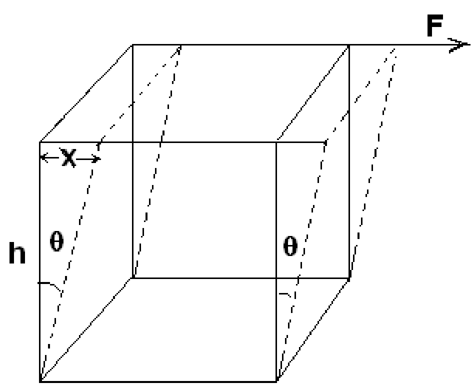
$$Bulk\ modulus\ (B) = Volume\ stress/\ Volume\ strain$$

$$B = -F/A\ /\ \Delta V/V = -FV/A\Delta V$$

Negative sign indicate reduction in volume The reciprocal of bulk modulus is called compressibility

$$compressibility = 1/\ bulk\ modulus$$

(iii) Modulus of rigidity: According to the definition, the ratio of shearing stress to shearing strain is called modulus of rigidity (η) . in this case the sphape of the body changes but its volume remains unchanged. Consider the case of a cube fixed at its lower face and acted upon by a tangential force F on its upper surface of area A as shown in figure



$$shearing\ stress = F/A$$

$$Shearing\ strain = \theta = x/h$$

$$\eta = F/A\theta = Fh/Ax$$

Solved Numerical

Q) A solid sphere of radius R made of a material of bulk modulus B is surrounded by a liquid in cylindrical container. A massless piston of area A flots on the surface of the liquid. Find the fractional change in the radius of the sphere (dR/R) when a mass M is placed on the piston to compress the liquid

Solution
From the formula of Bulk modulus

$$B = -FV/A\Delta V$$

$$V = 4/3\ \pi R^3$$

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$$dV = 4\pi R^2 dR$$

$$B = -F\ 4/3\ \pi R^3 / A 4\pi R^2 dR$$

$$dR\ R = Mg/\ 3AB$$

Poisson's Ratio:

It is the ratio of lateral strain to the longitudinal strain. For example, consider a force F applied along the length of the wire which elongates the wire along the length and it contracts radially. Then the longitudinal strain =Δl/l and lateral strain = Δr/r , where r is the radius of the wire

$$Poisson'\ s\ ratio\ (\sigma) = -\Delta r/r\ /\ \Delta l/l$$

$$\Delta r/\ r = -\sigma\ \Delta l/\ l$$

For rectangular bar: let b be breadth and h be thickness then

$$\Delta b/b = -\sigma\ \Delta l/l$$

$$\Delta h/h = -\sigma\ \Delta l/l$$

The negative sign indicates that change in length and radius is of opposite sign. Change in volume due to longitudinal force Due to application of tensile force, lateral dimension decreases and length increases. As a result there is a change in volume (usually volume increases). Let us consider the case of a cylindrical rod of length l and radius r.

Since $V=\pi r^2 L$

$$\therefore \Delta V/V = 2\ \Delta r/r + \Delta l/l\ (\ or\ very\ small\ change)$$

From above equations or radius and Length

$$\therefore \Delta V/V = -2\sigma\ \Delta l/l + \Delta l/l$$

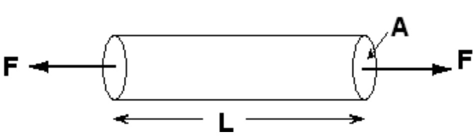
$$\therefore \Delta V/V = \Delta l/l\ (1 - 2\sigma)$$

Longitudinal Strain: $\varepsilon l = \Delta l/l$

$$\therefore \Delta V/V = \varepsilon l\ (1 - 2\sigma)$$

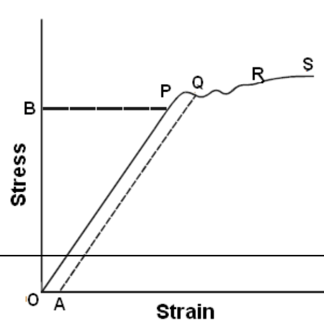
Above equation suggest that since Δv > 0, value of σ cannot exceed 0.5

Stress –Strain relationship for a wire subjected to longitudinal stress



Consider a long wire (made of steel) of cross-sectional area A and original length L in equilibrium under the action of two equal and opposite variable force F as shown in figure. Due to the application of force, the length gets changed to L +l. Then, longitudinal stress = F/A and Longitudinal strain = l/L

The extension of the wire is suitably measured and a stress – strain graph is plotted



(i) In the figure the region OP is linear. Within a normal stress, strain is proportional to the applied stress. This is Hooke's law. Up to P, when the load is removed the wire regains its original length along PO. The point P represents the elastic limit, PO represents the elastic range of the material and OB is the elastic strength.

(ii) Beyond P, the graph is not linear. In the region PQ the material is partly elastic and partly plastic. From Q, if we start decreasing the load, the graph does not come to O via P, but traces a straight line QA. Thus a permanent strain OA is caused in the wire. This is called permanent set.

(iii) Beyond Q addition of even a very small load causes enormous strain. This point Q is called the yield point. The region QR is the plastic range.

(iv) Beyond R, the wire loses its shape and becomes thinner and thinner in diameter and ultimately breaks, say at S. Therefore S is the breaking point. The stress corresponding to S is called breaking stress.

Elastic potential energy or Elastic energy stored in a deformed body

The elastic energy is measured in terms of work done in straining the body within its elastic limit Let F be the force applied across the cross-section A of a wire of length L. Let l be the increase in length. Then

$Y = F\ A\ l\ L = FL\ Al\ F = YAl\ L$

If the wire is stretched further through a distance of dl, the work done dw

$dW = F \times dl = YAl\ L\ dl$

Total work done in stretching the wire from original length L to a length L +l (i.e. from l = 0 to l = l)

$W = \int YAl\ L\ dl\ l\ 0$

$W = YA\ L\ l\ 2\ 2 = 1\ 2\ (AL)\ (\ l\ L\ )\ (\ l\ L\ )$

$W = 1\ 2 \times volume \times stress \times strain$

Solved Numerical

Q) The rubber cord of catapult has a cross-section area 1mm^2 and total unstrtched length 10 cm. It is stretched to 12cm and then released to project a body of mass 5g. taking the Young's modulus of rubber as 5×10^8 N/m^2 , calculate the velocity of projection

Solution

It can be assumed that the total elastic energy of catapult is converted into kinetic energy of the body without any heat loss L = 12cm = 12×10^{-2} m , l = 2cm = 2×10^{-3} m , A = 1mm^2 = 10^{-6}m

$$U = YA/\ ^2/2 = 5 \times 10^8 \times (1 \times 10^{-6}) \times (2 \times 10^{-2})^2 / 2 \times 10 \times 10^{-2} = 1$$

Now K.E of projectile = elastic energy of catapult

$$1/2\ mv^2 = U$$

$$1/2 \times 5 \times 10^{-3} \times v^2 = 1$$

V = 20 m/s

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