Fundamentals Of Relations And Functions

- Each element in a set is unique.
- The elements of a set may be discrete or continuous.
- A set may contain no element at all, finite number of elements or infinite number of elements.
- If A and B are two sets, then the following results hold true:

  - $\bar{\bar{A}}$ (or $(A^c)^c$) $= A$, $\quad A \cap A^c = \phi$ and $A \cup A^c = U$
  - $A \cup \phi = A$ and $A \cap \phi = \phi$
  - $A \cup U = U$ and $A \cap U = A$
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
  - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  - $\overline{A \cup B} = \bar{A} \cap \bar{B}$
  - $\overline{A \cap B} = \bar{A} \cup \bar{B}$

- Relation is a linear operation which establishes relationship between the elements of two sets according to some definite rule of relationship.
- If R is a relation from a set A to set B, then the set of all first components or coordinates of the ordered pairs is called the domain of R, while the set of all second components or coordinates of the ordered pairs is called as range of relation.
- If R is a relation defined from A to B, then R^{-1} is a relation defined from B to A as $R^{-1} = \{(b,a) | (a, b) \in R\}$.
- There can be various types of relations like:
  - Identity relation
  - Reflexive relation
  - Symmetric relation
  - Transitive relation
  - Equivalence relation
- A relation defined on a set A is said to be an identity relation if every element of A is related to itself and only itself.
- A relation such that $(a, b) \in R$ then $(a, a) \in R$ is reflexive. However, if there is a single ordered pair (a, b) $\in$ R, such that (a, a) does not belong to R, then R is not reflexive.
- A relation defined on a set is said to be symmetric if $a R b \Rightarrow b R a$.
- A relation defined on a set A is said to be transitive if a R b and b R c then a R c.
- A relation which is reflexive, symmetric as well as transitive is said to be an equivalence relation.

- A relation becomes a function iff:

  To each $a \in A$, there exists a unique 'b' $\in B$ such that $(a, b) \in f$.

  $(a_1, b_1) \in f$ and $(a_1, b_2) \in f \Rightarrow b_1 = b_2$.

- Let $f : A \rightarrow B$, then the domain and codomain of f are defined as

  Domain of f = $\{a \mid a \in A, (a, f(a)) \in f\}$

  Range of f = $\{f(a) \mid a \in A, f(a) \in B, (a, f(a)) \in f\}$

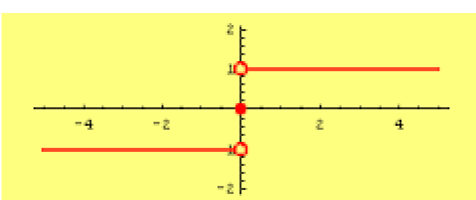
- **Basic Properties of |x|:**
  1. $||x|| = |x|$
  2. $|x| > a \Rightarrow x < a$ or $x < -a$ if $a \in R^+$ and $x \in R$ if $a \in R^-$
  3. $|x| < a \Rightarrow -a < x < a$ if $a \in R^+$ and no solution if $a \in R^- \cup \{0\}$
  4. $|x + y| \leq |x| + |y|$
  5. $|x - y| \geq |x| - |y|$
- The last two properties can be put in one compact form namely,
  1. $|x| - |y| \leq |x \pm y| \leq |x| + |y|$
  2. $|xy| = |x|\,|y|$
  3. $|x/y| = |x/y|$, $y \neq 0$
- For 'y' to be an algebraic function of x it should satisfy an algebraic expression of the form $P_0(x)y^n + P_1(x)y^{n-1} + \ldots.. + P_{n-1}(x)y + P_n(x) = 0$
- All polynomial functions are algebraic but every algebraic function need not be a polynomial function.
- **Signum Function:**

  A function y = f(x) = Sgn (x) is defined as follows:

  y = f(x) = 1 for x > 0

  = 0 for x = 0

  = -1 for x < 0



- **Properties of greatest integer function:**
    1. $[x] \le x < [x] + 1$
    2. $x-1 < [x] \le x$
    3. $[x+m] = [x] + m$ if m is an integer
    4. $[x] + [y] \le [x + y] \le [x] + [y] + 1$
    5. $[x] + [-x] = 0$ if x is an integer

       $= -1$ otherwise
- Range of y = f(x) is the collection of all outputs corresponding to each real number of the domain. To find the range of a function:
    1. Find the domain of the function y = f(x) first.
    2. If domain is a set having only finite number of points, then range is the set of corresponding f(x) values.
    3. If domain of y = f(x) is R or R- {some finite points}, then express x in terms of y. from this, find y for x to be defined or form an equation in terms of x and apply the condition for real roots.
- Two functions 'f' and 'g' are identical if and only if they satisfy the following conditions:
    1. Domain of f = Domain of g
    2. Range of f = Range of g
    3. $f(x) = g(x)$ $\forall x$ belonging to the common domain
- If a function is entirely increasing or decreasing in some entire domain, then the function is one-to one.
- If a line parallel to the x-axis cuts the graph of the function at a single point, then the function is one-to-one.
- If a continuous function has at least one local maximum or local minimum, then the function is many-one.
- A function can either be one-one or many-one but not both.
- If range of a function f(x) = codomain of f(x), then the function f(x) is onto.
- A polynomial function of even degree is always an into function.
- A function can only be one of these four types:
    1. One-one onto (injective and surjective)
    2. One –one into (injective but not surjective)
    3. Many-one onto (surjective but not injective)
    4. Many-one into (neither injective nor surjective)

- A continuous function which is always increasing or decreasing in the whole domain is one-one.
- A function is one to one iff a horizontal line intersects its graph at most once.
- If the range and co-domain for a function are same then it is onto.
- If a set A contains ‘n’ distinct elements then the number of possible distinct functions from A to A is n^n and out of these n! are one-one.
- The range of a constant function is always a singleton.
- A constant function may be one-one, many-one, onto or into.
- If ‘f’ and ‘g’ are real valued functions with domains as A and B respectively, then both f and g are defined in A∩B.
    1. The domain of f + g, f-g and f.g is also A ∩ B.
    2. The domain of (f/g) is {x | x ∈A ∩ Bs.t. g(x) ≠ 0}.
- For the product ‘gof’ of two functions f and g, the range of f must be a subset of the domain of g.
- The composite of functions is not commutative, i.e. gof ≠ fog.
- The composite of functions is associative i.e. if f, g and h are three functions such that fo(goh) = (fog)oh.
- The composite of two bijections is a bijection i.e. if f and g are two bijections such that gof is defined then gof is also a bijection.
- The inverse of a bijection is unique.
    1. If f: A→ B is a bijection, and g:B→ A is its inverse, then fog = I_B and gof = I_A, where I_A and I_B are the identities of A and B respectively.
    2. The graphs of f and g in this case are the mirror images of each other in the line y = x.
    3. The inverse of a bijection is also a bijection.
    4. If f and g are two bijections such that f: A→ B is a bijection, and g: B→ C, then the inverse of gof also exists and $(fog)^{-1} = f^{-1}og^{-1}$
- The function is said to be even if f(x)-f(-x) = 0 and the function is said to be odd if f(x) + f(-x) = 0
    1. A function may neither be even nor odd.
    2. Inverse of an even function is not defined.
    3. Every even function is symmetric about the y-axis and every odd function is symmetric about origin.
    4. It is possible to express every function as the sum of an even and odd function i.e.

       f(x) = [{f(x) + f(-x)}/2 + {f(x)-f(-x)}/2], where the first part is an even function and the second part is the odd function.
    1. If function f and g are both even or both odd then the function f.g will be even but if either of them is odd then f.g will be odd.

2. The only function which is defined on the entire number line and which is even as well as od at the same time is f(x) = 0.

- The inverse of a periodic function does not exist.

1. Every constant function is always periodic but with no fundamental period.
2. If a function f(x) has period ‘T’ and the function g(x) also has the same period, then it is not necessary that the function f(x) + g(x) will also have ‘T’ as its period.
3. If the function f has ‘p’ as its period then 1/f(x) and √f(x) also have p as the period.

- If f(x) has period ‘T’ then f(ax+b) has a period T/a (a > 0).
- Logarithm of a number to some base is the exponent by which the base must be raised in order to get that number.
- Logarithm of zero does not exist and logarithm of negative reals is not defined in the system of real...

2. Methods of representing a set

For sets, we simply put each element, separated by a comma, and then put some curly brackets around the whule thing.
Sets are usually denoted by capital letters A, B, C, X, Y, Z, etc
The elements of a set are represented by small letters *a, b, c, x, y, z,* etc
When we say an element a is in a set A, we use the symbul ∈ to show it.
And if something is not in a set use ∉
Example: In a set of even number E, 2∈E but 3∉E
Two Methods are used to represent Sets

(a) Roster forms

In a Roster forms, all the elements in the set is listed.
Example
Set of Vowel ={ a,e,i,o,u}
Some Important points

In roster form, the order in which the elements are listed is immaterial

- while writing the set in roster form an element is not generally repeated

(b) Set Builder Form

- In set-builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set.For example, in the set {2,4,6,8}, all the elements possess a common property, namely, each of them is a even number less than 10. Denoting this set by N, we write N={x:xisaevennumberless10}

- We describe the element of the set by using a symbul x (any other symbul like the letters *y, z,* etc. could be used) which is fullowed by a culon ":" . After the sign of culon, we write the characteristic property possessed by the elements of the set and then enclose the whule description within braces

3. Types of sets

(a) Empty set

- A set which does not contain any element is called the *empty set* or the *null set* or the *void set*
- It is denoted by ϕ or $\{\}$
- It is a set with *no elements*
- Examples of empty sets is
  $D=\{x:x^2=9, x is even\}$
  Here D is the empty set, because the equation $x^2=9$ is not satisfied by any even value of x

(b) Finite or infinite set

- If M is a set then $n(M)$ defines the number of distinct elements in the set M.
- If $n(M)$ is zero or finite ,then M is a finite set
- If $n(M)$ is infinite then M is a infinite set

(c) Equal sets

- Two sets are said to be equal if they have same members in them.
- For A and B to be equal, every member of A should be present in set B and every member of $B$to be present in set A
- It is denoted by equality sign $A=B$

4. Subset and Proper Subset

- A set A is said to be a subset of a set B if every element of A is also an element of B .
- It is denoted by
  $A\subset B$ if whenever $a\in A$, then $a\in B$
- If $A\subset B$ and $B\subset A$, then $A=B$.
- Every set is subset of itself $A\subset A$
- Empty set is subset of every set $\phi\subset A$
- If $A\subset B$ and $A\neq B$, then A is proper subset of B. In such a case B is called *superset* of set A

5. Subset of set of the real numbers

N : the set of all natural numbers
Z : the set of all integers
Q : the set of all rational numbers
R : the set of real numbers
Z+ : the set of positive integers
Q+ : the set of positive rational numbers, and
R+ : the set of positive real numbers
$T=\{x:x\in R and x\notin Q\}$, i.e., all real numbers that are not rational
$N\subset Z\subset Q, Q\subset R, T\subset R, N\not\subset T$

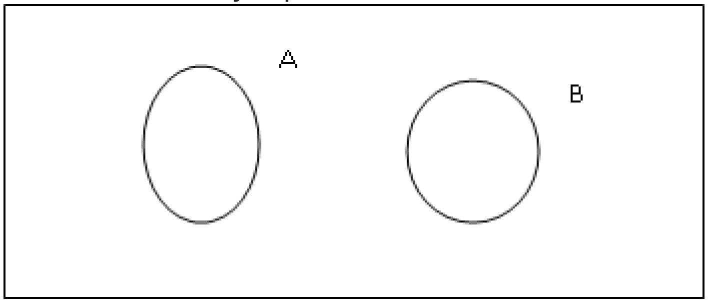
6. Interval as subset of R Real Number

$a,b\notin R, b>a$

(a,b)	It is the open interval set between point and b such that All the points between a and b belong to the open interval *(a, b)* but *a, b* themselves do not belong to this interval	$\{y:a<y<b\}$
[a,b]	It is the closed interval set between point and b such that All the points between *a* and *b* belong to the open interval *(a, b)* including *a, b*	$\{x:a\le x\le b\}$
[a,b)	It is the open interval set between point and b such that All the points between*a* and *b* belong to the open interval *(a, b)* including *a,* but not *b*	$\{x:a\le x<b\}$
(a,b)	It is the open interval set between point and b such that All the points between*a* and *b* belong to the open interval *(a, b)* including *b*, but not *a*	$\{x:a<x\le b\}$

Venn diagram

Venn diagrams were introduced in 1880 by John Venn (1834–1923). These diagrams consist of rectangles and closed curves usually circles. The universal set is represented usually by a rectangle and its subsets by circles. Venn diagrams normally comprise overlapping circles. The interior of the circle symbolically represents the elements of the set, while the exterior represents elements that are not members of the set. For instance, in a two-set Venn diagram, one circle may represent the group of all wooden objects, while another circle may represent the set of all tables

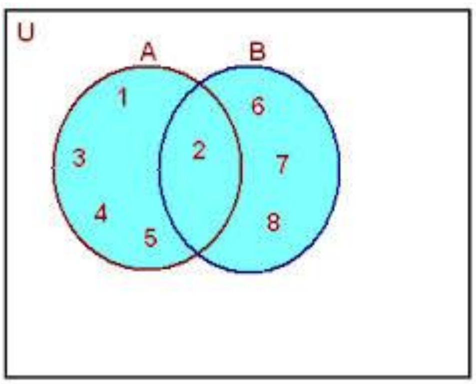


Operation on Sets

Union of Sets

The union of two sets A and B is the set C which consists of all those elements which are either in Aor in B (including those which are in both). In symbols, we write.
$A \cup B = \{x : x \in A or x \in B\}$

Venn Digram



A Union B

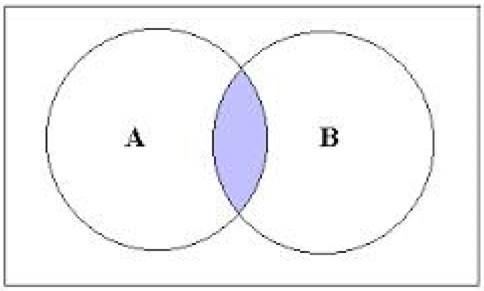
Some Properties of the Operation of Union

- Commutative law : $X \cup Y = Y \cup X$
- Associative law : $(X \cup Y) \cup Z = X \cup (Y \cup Z)$
- Law of identity element, ϕ is the identity of ∪ : $X \cup \phi = X$
- Idempotent law : $X \cup X = X$
- Law of U : $U \cup X = U$

Intersection of Sets

The Intersection of two sets A and B is the set C which consists of all those elements which are present in both A and B . In symbols, we write.
$A \cap B = \{x : x \in A and x \in B\}$

Venn Digram



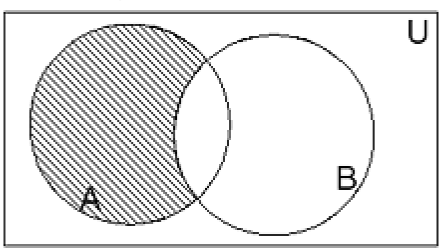
Some Properties of Operation of Intersection

- Commutative law : $X \cap Y = Y \cap X$
- Associative law : $(X \cap Y) \cap Z = X \cap (Y \cap Z)$
- Law of $\cap$ and U : $\phi \cap X = \phi$, $U \cap X = X$
- Idempotent law : $X \cap X = X$
- Distributive law : $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

Difference of set

The difference of two sets A and B is the set C which consists of all those elements which are present in A but not in B . In symbols, we write,
$A - B = \{x : x \in A and x \notin B\}$

Venn Digram



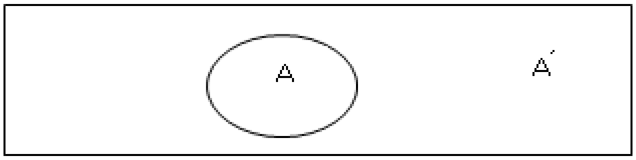
Some Properties of Operation of Difference

- $A - B \neq B - A$
- The sets $(A - B)$, $(A \cap B)$ and $(B - A)$ are mutually disjoint sets.

Compliment of set

Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of UU which are not the elements of A. Symbolically, we write A' to denote the complement of $A$with respect to U. Thus,
$A' = \{x : x \in U and x \notin A\}$,obviously $A' = U - A$

Venn Digram



Some Properties of compliment_of_sets

1. Complement laws:
   - $A \cup A' = U$
   - $A \cap A' = \phi$
2. De Morgan's law:
   - $(A \cup B)' = A' \cap B'$
   - $(A \cap B)' = A' \cap B'$
3. Law of double complementation: $(A')' = A$
4. Laws of empty set and universal set : $\phi' = U$ and $U' = \phi$

Cardinality of the set

- The cardinality of the set defines the number of element in the Set
- If A is the set, Cardinality of the set is defined as $n(A)$
- For $A = \{1,2,3\}$ then $n(A) = 3$

Set Relations

Joined Set	Disjoined Set
Set having common elements	Set having no common elements
$n(X \cap Y) \neq 0$	$n(X \cap Y) = 0$

Important Operation on Cardinality

1. If $n(X \cap Y) \neq 0$
   $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$
2. If $n(X \cap Y) = 0$
   $n(X \cup Y) = n(X) + n(Y)$