Logarithms

THEORY

Let a be a positive real number, a ≠ 1 and a^x = m. Then x is called the logarithm of m to the base a and is written as $\log_a m$, and conversely, if $\log_a m = x$, then $a^x = m$.

Two Properties of Logarithms

1. $\log_a 1 = 0$ for all a> 0, a ≠ 1
   That is, log 1 to any base is zero
   Let $\log_a 1 = x$. Then by difference, $a^x = 1$
   But $a^0 = 1 \therefore a^x = a^0 \Leftrightarrow x = 0$
   Hence $\log_a 1 = 0$ for all a > 0, a ≠ 1

Laws of Logarithms

First Law: $\log_a (mn) = \log_a m + \log_a n$

That is, log of product = sum of logs

Second Law: $\log_a (m/n) = \log_a m - \log_a n$

That is, log of quotient

= difference of logs

The Characteristic and Mantissa of a Logarithm

The logarithm of a number consists of two parts: the integral part and the decimal part. The integral parts is known as the characteristic and the decimal part is called the mantissa.

Base Change Rule

This rule states that

i. $\log_a (b) = \log_c (b)/\log_c (a)$

It is one of the most important rules for solving lagarithms.

ii. $\log_b (a) = \log_c (a)/\log_b (c)$

A corollary of this rule is

iii. $\log_a (b) = 1/\log_b (a)$

iv. log c to the base a^b is equal to log a^c/b

Results on Logarithmic Inequalities

a) If $a > 1$ and $\log_a x_1 > \log_a x_2$ then $x_1 > x_2$
b) If $a < 1$ and $\log_a x_1 > \log_a x_2$ then $x_1 < x_2$

Applied conclusions for logarithms

1. The characteristic of common logarithms of any positive number less than 1 is negative
2. The characteristic of common logarithm of any number greater than 1 is positive
3. If the logarithm to any base a gives the characteristic n, then we can say that the number of integers possible is given by $a^{n+1} – a^n$.