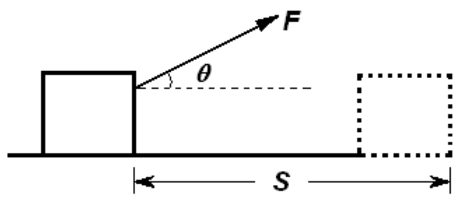
WORK, ENERGY AND POWER

Work done by a constant force:

Consider an object undergoes a displacement S along a straight line while acted on a force



F that makes an angle θ with S as shown The work done W by the agent is the product of the component of force in the direction of displacement and the magnitude of displacement W = FScosθ Work done is a scalar quantity and S.I. unit is N-m or Joule (J). Its dimensional formula is $M^1 L^2 T^{-2}$ We can also write; work done as a scalar product of force and displacement

$$W = F \cdot S$$

From this definition, we conclude the following points:

(i) Force does not work if point of application of force does not move (S=0)

(ii) Work done by a force is zero if displacement is perpendicular to the force ($\theta=90^O$)

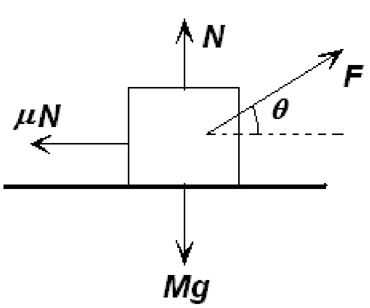
(iii) If angle between force and displacement is acute (θ 90^O), we say that work done by the force is negative or work is done by the object

(iv) If angle between force and displacement is obtuse ($\theta > 90^O$), we say that work done by the force is negative or work is done by the object

Solved Numerical

Q) A block of mass M is pulled along a horizontal surface by applying a force at an angle θ with horizontal. Coefficient of friction between block and surface is µ. If the block travels with uniform velocity, find the work done by this applied force during a displacement d of the block

Solution



The forces acting on the block as shown in figure Force F will resolve as Fsinθ along normal while Fcosθ will be opposite to friction.

Thus we get Fcosθ = µN ----- (1)

And N + Fsinθ = Mg -----eq(2)

Eliminating N from equation (1) and (2) Fcosθ = µ (Mg - Fsinθ)

F = µMg /cosθ + sinθ

Work done by this force during displacement d

$$W = Fd = \mu Mgd/ \cos\theta + \sin\theta$$

Power

If external force is applied to an point like object and if the work done by this force is ΔW in the time interval Δt, then the average power during this interval is defined as $P = \Delta W\ \Delta t$ The work done on the object contributes to increasing energy of the object. A more general definition of power is the time rate of energy transfer. This instantaneous power is the limiting value of the average power as Δt approaches zero

$$P = dW/\ dt$$

Where we have represented the infinitesimal value of the work done by dW (even though it is not a change and therefore not a differential). We find from equation (2) that

$$dW = \vec{F} \cdot \vec{S}$$

Therefore the instantaneous power can be written as

$P = dW/\ dt = \vec{F} \cdot d\vec{S}\ dt = \vec{F} \cdot \vec{v}$

The SI unit of power is Joule per second (J/s), also called watt (W) 1W = 1 J/s = 1kgm2 s -3

Energy

A body is said to possess energy if it has the capacity to do work. When a body possessing energy does some work, part of its energy is used up. Conversely if some work is done upon an object, the object will be given some energy. Energy and work are mutually convertible.

Kinetic energy

Kinetic energy (K.E.) is the capacity of a body to do work by virtue of its motion If a body of mass m has velocity v its kinetic energy is equivalent to the work, which an external force would have to do to bring the body from rest to its velocity v. The numerical value of the kinetic energy can be calculated from the formula

$$K.E. = 1\ 2\ mv^2$$

This formula can be derived as follows:
Consider a constant force F which acting on a mass m initially at rest, particle accelerate with constant velocity and attend velocity v after displacement of S .
For the formula
v 2 – u 2 = 2as
Initial velocity is zero
v 2 = 2as
Multiply both the sides by m
mv2 = 2mas
mv2 = 2W [As work = FS = mas]

W = (1/2)mv2

But Kinetic energy of body is equivalent to the work done in giving the velocity to the body Hence K.E = (1/2)mv2

Since both m and v2 are always positive K.E is always positive and does not depend up on the direction of motion of body. Another equation for kinetic energy

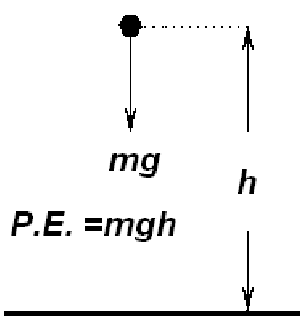
$$E = 1/2\, mv^2 = 1/2\, m^2v^2\, m = 1/2\, p^2/\text{m}$$

Potential energy

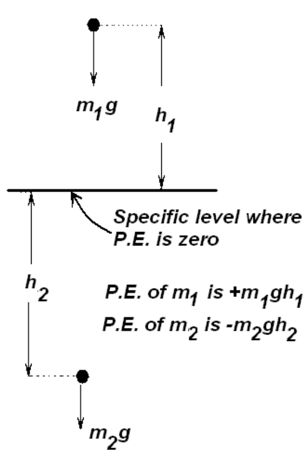
Potential energy is the energy due to position. If a body is in a position such that if it were released it would begin to move, it has potential energy There are two common forms of potential energy, gravitational and elastic

Gravitational potential energy

It is possessed by virtue of height When an object is allowed to fall from one level to a lower level it gains speed due to gravitational pull, i.e. it gains kinetic energy. Therefore, in possessing height, a body has the ability to convert its gravitational potential energy into kinetic energy.



The gravitational potential energy is equivalent to the negative of the amount of work done by the weight of the body in causing the descent. If a mass m is at a height h above a lower level, the P.E. possessed by the mass is (mg) (h) Since h is the height of an object above a specific level, an object below the specified level has negative potential energy



Therefore Gravitational Potential Energy = ± mgh

- The chosen level from which height is measured has no absolute position. It is important therefore to indicate clearly the zero P.E. level in any problem in which P.E. is to be calculated.

- Gravitational Potential Energy = ± mgh is applicable only when h is very small in comparison to the radius of earth.

Elastic potential Energy

It is a property of stretched or compressed springs. The end of a stretched elastic spring will begin to move if it is released. The spring therefore possesses potential energy due to its elasticity (i.e. due to change in its configuration)

The amount of elastic potential energy stored in a spring of natural length a and spring constant k when it is extended by a length x is equal to the amount of work necessary to produce the extension

Work done = $(1/2)kx^2$ so
Elastic Potential energy = $(1/2)\ kx^2$
Elastic potential energy is never negative whether the spring is extended or compressed

Work energy theorem

When a body is acted upon by force acceleration is produced in it. Thus velocity of the body changes and hence the kinetic energy of the body also changes. Also force acting on a body displaces the body and so work is said to be done on the body by force. These facts indicate that there should be some relation between the work done on body and change in its kinetic energy.
The work done by the force F
W= F S
W = ma
W = mas
Also $v^2 - u^2 = 2as$
Multiplying both sides by m.
$m(v^2 - u^2) = 2ams$

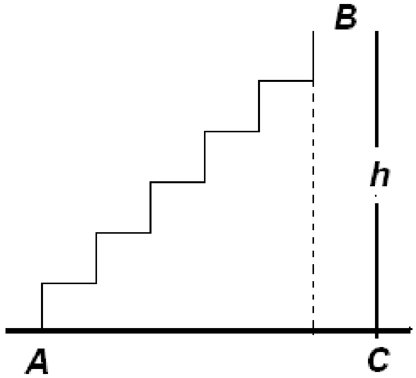
$$1/2\ mv^2 - 1/2\ u^2 = mas$$
$$1/2\ mv^2 - 1/2\ u^2 = W$$

Here u and v are the speeds before and after application of force. The left hand side of above equation gives change in kinetic energy while right hand gives the work done
Thus $\Delta K = W$
The work done by the resultant force on a body is equal to change in kinetic energy of the body. This statement is known as work energy theorem.

Conservative and non-conservative force

Conservative force

A conservative force may be defined as on for which work done in moving between two points A and B is independent of the path taken between two points. Work done to move particles through stairs is equal to moving particle vertically .The implication of “conservative” in this context is that you could move it from A and B by one path and returns to A by another path with no net loss of energy – any close return path A takes net work zero. Or mechanical energy is conserved



A further implication is that the energy of an object which is subject only to that conservative force is dependent upon its position and not upon the path by which it reached that position. This makes it possible to define a potential energy function which depends upon position only If a force acting on an object is a function of position only, it is said to be a conservative force and it can be represented by potential energy function which for a one-dimensional case satisfies the derivative condition

$$F(x) = -\, dU/dx$$

Example for verification

(a) Gravitational potential energy = -mgh
Thus

$$F(h) = -\, d(-mgh)\,/dt$$

F(h) = mg

(b) Spring potential energy = (1/2)kx^2

$$(x) = -\,1/\,2\;k\;d(x^2\,)/\,dt$$
$$F(x) = -kx$$

Non-conservative force

Consider a body moving on a rough surface from A to B and then back from B to A. Work done against frictional forces only add up because in both the displacement work is done against frictional force only. Hence frictional force cannot be considered as a conservative force. It is non-conservative force

Conservation of mechanical energy

Kinetic and potential energy both are forms of mechanical energy. The total mechanical energy of a body or system of bodies will be changed in values if

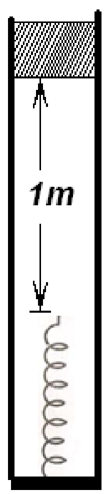
(a) An external force other than weight causes work to be done(work done by weight is potential energy and is therefore already included in the total mechanical energy)
(b) Some mechanical energy is converted into another form of energy (e.g. sound, heat , light) such a conversion of energy usually takes place when a sudden change in the motion of the system occurs. For instance, when two moving objects collide some mechanical energy is converted into sound energy, which is heard as a bang at the impact.

If neither (a) nor (b) occurs, then the total mechanical energy of a system remains constant. This is the principle of Conservation of Mechanical Energy and can be expressed as The total mechanical energy of a system remains constant provided that no external work is done and no mechanical energy is converted into another form of energy When this principle is used in solving problems, a careful appraisal must be made of any external forces, which are acting. Some external forces do work and hence cause a change in the total energy of the system.

Solved Numerical

Q) A 20kg body is released from rest, so as to slide in between vertical rails and compresses a spring having a force constant k = 1920 N/m. the spring is 1m below the starting position of the body. The rail offers a resistance of 36N to the motion of the body. Find (i) the velocity of the body just before touching the spring (ii) the distance ,l through which the spring is compressed (iii) the distance 'h' through which the body rebounds up Solution

Solution



(i) Let velocity of the body just before touching the spring be v Change in k.E = work done

$$1/2\ mv^2 - 0 = mg \times 1 - 36 \times 1$$
$$1/2 \times 20 \times v^2 = 20 \times 9.8 \times 1 - 36 \times 1$$
$$v = 4\ m/s$$

(ii) Let x be maximum compression of the spring. Then effective height for calculation of potential energy = 1+x From conservation of energy Spring potential energy = Change in P.E - Work done against friction

$$1\ 2\ kx^2 = m(1 + x) - 36(1 + x)$$
$$1/2 \times 1920 \times x^2 = 20 \times 9.8 \times (1 + x) - 36(1 + x)$$
$$X = 0.5\ m$$

(iii) Let object bounce up to height h Potential energy of object = spring potential energy – work against friction

$$mgh = 1\ 2\ kx^2 - 36h$$
$$20 \times 9.8 \times h = 1/2 \times 1920 \times (0.5)^2 - 36h$$
$$h = 240\ 232 = 30\ 29 = 1.03m$$